Computing the execution probability of jobs with replication in Mixed-Criticality schedules

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Static non-preemptive Mixed-Criticality scheduling

We treat MC scheduling as a discrete stochastic optimization problem:

- **Environment:** time-triggered, non-preemptive
- **Given:** finite set of jobs with known criticalities but unknown execution times
- **Goal:** compute offline schedule satisfying the given constraints (releases, deadlines, resources, ...)
- **The main trick:** schedule contains alternatives – online execution compensates for the observed realizations of execution times

Motivation for static environments:

- advantages: predictability, certification, ...
- disadvantages: rigid, less flexible, scheduling algorithms, ...

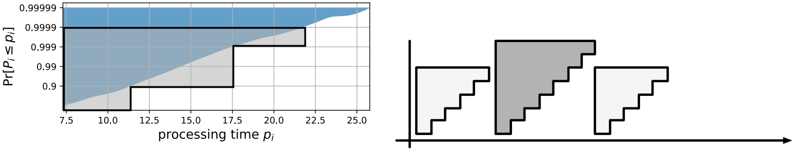
→ mitigate some of the disadvantages

Model of MC job

Mixed-criticality job: Let $T_i = (\boldsymbol{\pi}_i, \boldsymbol{\mu}_i, \mathcal{X}_i)$ be a mixed-criticality job, where

- $\mathcal{X}_i \in \mathbb{N}$ is a positive integer criticality
- $\boldsymbol{\pi}_i = \left(\pi_i^{(1)}, \ldots, \pi_i^{(\mathcal{X}_i)}\right) \in \mathbb{N}^{\mathcal{X}_i}$ is a vector of non-decreasing execution times of the job
- $\boldsymbol{\mu}_i = \left(\mu_i^{(1)}, \ldots, \mu_i^{(\mathcal{X}_i)}\right) \in [0, 1]^{\mathcal{X}_i}$ is a conditional probability distribution over $\boldsymbol{\pi}_i$ given that T_i is executed.

How to obtain it?



(a) Approximation of a CDF. (b) Message retransmissions – triangles.

Novák, A.; Šůcha, P.; Hanzálek, Z. "*Scheduling with uncertain processing times in mixed-criticality systems*" European Journal of Operational Research. 2019, 279(3), 687-703. ISSN 0377-2217.

Durr, Ch. et al. "*The triangle scheduling problem*" Journal of Scheduling. 2018, 21(3), 305-312. ISSN 1094-6136.

Runtime execution of static MC schedules

How the schedules are evaluated?

→ problem with rigidity of time-triggered environments: a message scheduling example

Functionality: brakes T_1, engine T_2, T_4, T_5, infotainment T_3

- A static schedule assigning start times to messages:

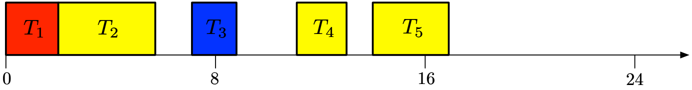


Figure: No retransmissions assumed.

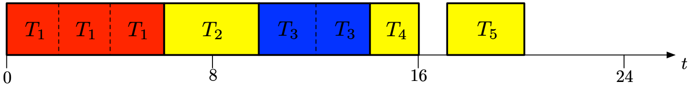


Figure: More time allocated for retransmissions.

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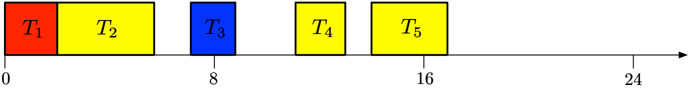

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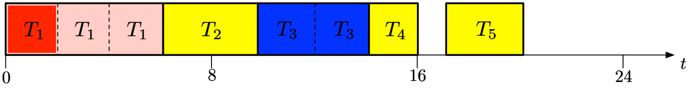


Figure: More time allocated for retransmissions.

Runtime execution of static MC schedules

How the schedules are evaluated?
→ problem with rigidity of time-triggered environments: a message scheduling example

Solution: static schedules robust with respect to execution time prolongation

- each message has a different commitment to retransmission
- the execution policy skips less critical messages to compensate for the retransmisssions of more critical ones
- a trade-off between efficient use of resources and safety margins

Runtime execution of static MC schedules

Functionality: brakes T_1, engine T_2, T_4, T_5, infotainment T_3

Static non-preemptive time-triggered MC schedule:

- A schedule that accounts for **alternative execution scenarios**
- Basic requirement: no overlap at any criticality level

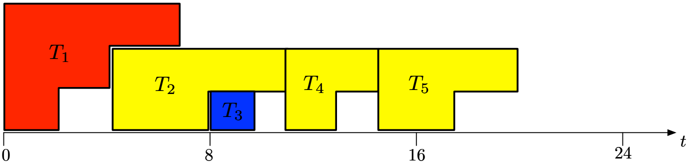


Figure: A static schedule with mixed-criticality jobs.

Novák, A.; Šůcha, P.; Hanzálek, Z. *"Efficient Algorithm for Jitter Minimization in Time-Triggered Periodic Mixed-Criticality Message Scheduling Problem"*, *RTNS'16*, p. 23-31. ISBN 978-1-4503-4787-7.

Runtime execution of static MC schedules

Best-case scenario.

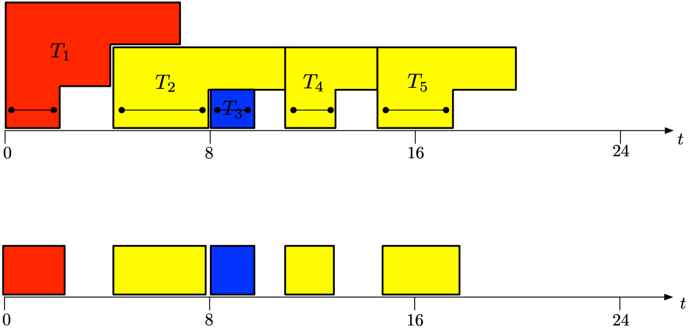


Figure: Realized schedule during scenario $e^{(1)}$.

Novák, A.; Šůcha, P.; Hanzálek, Z. "*Efficient Algorithm for Jitter Minimization in Time-Triggered Periodic Mixed-Criticality Message Scheduling Problem*", *RTNS'16*, p. 23-31. ISBN 978-1-4503-4787-7.

Runtime execution of static MC schedules

T_1 rejects T_2, then the system mode is decreased.

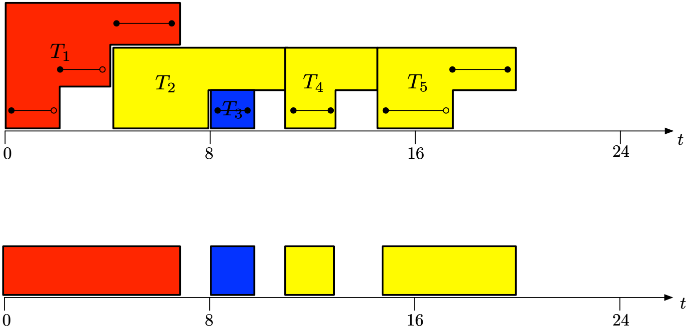
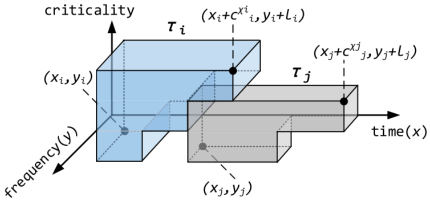


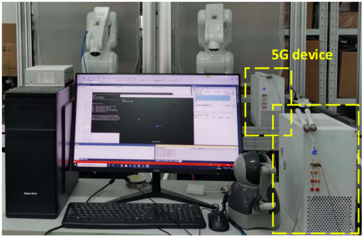
Figure: Realized schedule during scenario $e^{(2)}$.

Novák, A.; Šůcha, P.; Hanzálek, Z. *"Efficient Algorithm for Jitter Minimization in Time-Triggered Periodic Mixed-Criticality Message Scheduling Problem"*, *RTNS'16*, p. 23-31. ISBN 978-1-4503-4787-7.

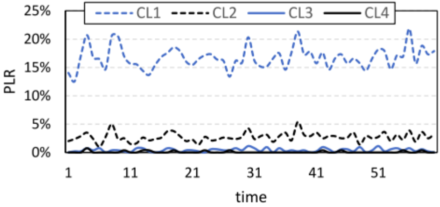
Application: 5G NR industrial message scheduling



(a) Frequency-enhanced model of MC job.



(b) 5G testbed.



(c) Packet loss for different criticalities.

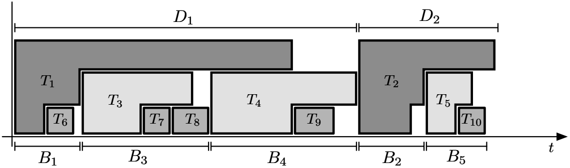
Jin, Xi et al., "*Mixed-Criticality Industrial Data Scheduling on 5G NR*" IEEE Internet of Things Journal, vol. 9, no. 12, 2022. ISSN 2327-4662.

Optimization problems: Period length minimization

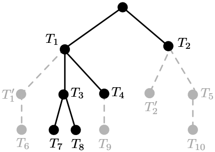
How we should obtain the schedule of MC jobs?

Different optimization problems:

- Period length minimization creates "packed" schedules [NOV2019]



(a) Minimizes schedule length.



(b) Critical subtree.

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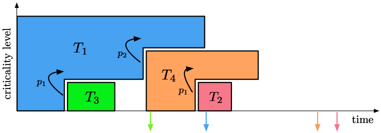
Optimization problems: Execution probability maximization

Different optimization problems:

- Period length minimization creates "packed" schedules [NOV2019]
- Execution probability favours more "spread" schedules [SED2017]



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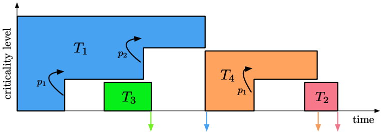


(b) Maximizes execution probability.

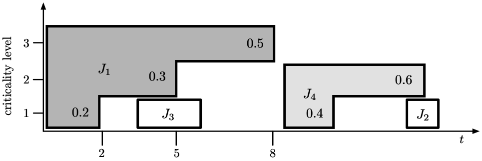
- (scheduling) Fixed order of jobs, 2 criticality levels: weakly NP-hard, general problem is strongly NP-hard
- (analysis) Computation of execution probability: polynomial time, closed-form solution

Seddik, Y.; Hanzálek, Z. "*Match-up scheduling of mixed-criticality jobs: maximizing the probability of jobs execution*" European Journal of Operational Research. 2017, 262(1), 46-59. ISSN 0377-2217.

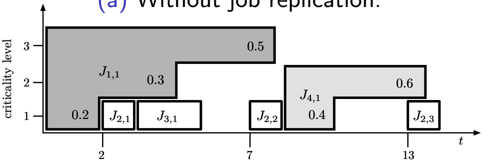
Job replication: fill in the empty slots

Replication: fill empty slots if possible:

- Motivation: improve flexibility by allowing alternative start times (slots) [NOV2022], appears as a replicated job in a schedule



(a) Without job replication.



(b) With job replication: J_2 has three replicas.

Novák, A.; Hanzálek, Z. "*Computing the execution probability of jobs with replication in mixed-criticality schedules*" Annals of Operations Research. 2022, 309(1), 209-232. ISSN 0254-5330.

The price of the replication

Replication has its price:

- improved flexibility $\rightarrow$ increased computational complexity
- (analysis) deciding $P_i > 0$ in a given schedule is NP-complete

Proof idea: A reduction from 3-SAT.

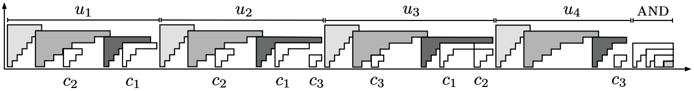


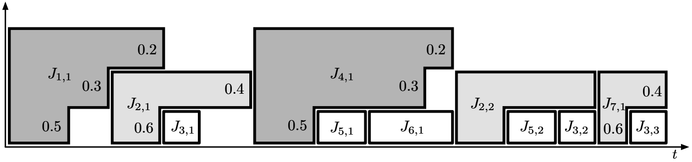
Figure: Schedule of the reduction from
$\mathcal{T} = c_1 \wedge c_2 \wedge c_3 = (u_1 \vee u_2 \vee u_3) \wedge (\neg u_1 \vee \neg u_2 \vee u_3) \wedge (u_2 \vee \neg u_3 \vee u_4)$.

Notes:

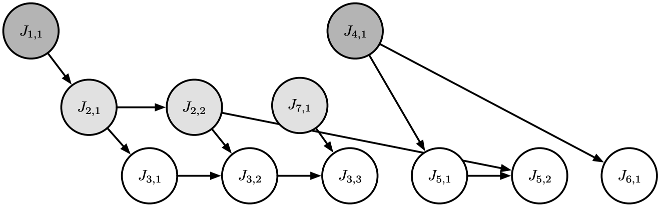
- problem remains hard if either of maximum number of criticality levels $\mathcal{L}$ or maximum number of replicas $\mathcal{R}$ is unbounded
- computing the precise value P_i is at least as hard as computing the number of satisfying assignments of a 3-CNF formula (i.e., #P-hard)

How to calculate the execution probability?

Main idea: represent the schedule as a Bayesian network – probabilistic graphical model.



(a) Example schedule $\boldsymbol{s}$.



(b) Corresponding Bayesian network $G(\boldsymbol{s})$.

Figure: Representation of a schedule by a Bayesian network.

How to calculate the execution probability?

Main idea: represent the schedule as a Bayesian network – probabilistic graphical model.

- Conditional probability tables (CPT)
- Use existing algorithms for inference in Bayesian networks

outcome					evidence
$\dagger$	$\star$	1	2	3	
0.0	0.0	0.5	0.3	0.2	$\emptyset$

(a) Replicas $J_{1,1}$ and $J_{4,1}$.

outcome				evidence
$\dagger$	$\star$	1	2	
0.0	0.0	0.6	0.4	$\emptyset$

(b) Replica $J_{7,1}$.

outcome				evidence
$\dagger$	$\star$	1	2	$J_{1,1}$
0.0	0.0	0.6	0.4	$\dagger \vee \star \vee 1 \vee 2$
0.0	1.0	0.0	0.0	3

(c) Replica $J_{2,1}$.

outcome				evidence
$\dagger$	$\star$	1	2	$J_{2,1}$
1.0	0.0	0.0	0.0	$\dagger \vee 1 \vee 2$
0.0	0.0	0.6	0.4	$\star$

(d) Replica $J_{2,2}$.

outcome			evidence
$\dagger$	$\star$	1	$J_{2,1}$
0.0	0.0	1.0	$\dagger \vee \star \vee 1$
0.0	1.0	0.0	2

(e) Replica $J_{3,1}$.

outcome			evidence	
$\dagger$	$\star$	1	$J_{3,1}$	$J_{2,2}$
1.0	0.0	0.0	$\dagger \vee 1$	
0.0	0.0	1.0	$\star$	$\dagger \vee \star \vee 1$
0.0	1.0	0.0	$\star$	2

(f) Replica $J_{3,2}$.

Figure: CPTs for the Bayesian network.

Conclusion

- non-preemptive time-triggered model of MC with uncertain execution times
- time-triggered MC schedules with alternative execution scenarios
- different optimization problems: makespan, jitter, **execution probability maximization with job replication**
- flexibility can be expensive $\rightarrow$ job replication introduces a new level of complexity: computation of the execution probability becomes hard
- modeling the schedules via Bayesian networks, the inference is efficient when both the maximum number of criticality levels $\mathcal{L}$ and replicas $\mathcal{R}$ remain constant

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